arXiv:nucl-th/9310029v1 22 Oct 1993

Deep Inelastic Scattering of Polarized Electrons off Polarized ^{3}He: Nuclear Effects and the Neutron Spin Structure Functions [1]

C. CIOFI degli ATTI
Dipartimento di Fisica, Università di Perugia,
and
INFN, Sezione di Perugia, Via A. Pascoli, I-06100 Perugia, Italy

E. PACE
Dipartimento di Fisica, Università di Roma "Tor Vergata",
and
INFN, Sezione Tor Vergata, Via E. Carnevale, I-00173 Roma, Italy

G. SALMÈ
INFN, Sezione Sanità, Viale Regina Elena 299, I-00161Roma, Italy

S. SCOPETTA
Dipartimento di Fisica, Università di Perugia,
and
INFN, Sezione di Perugia, Via A. Pascoli, I-06100 Perugia, Italy

Abstract

Nuclear effects in Deep Inelastic Scattering of polarized electrons off polarized ^{3}He are analyzed in terms of a spin dependent spectral function taking into account S' and D waves in ^{3}He, as well as Fermi motion and binding effects. A simple and reliable equation relating the neutron and ^{3}He spin structure functions is proposed.

The spin structure functions (SSF) of the nucleon g_1^N and g_2^N provide information on the spin distribution among the nucleon partons and can allow important tests of various models of hadron's structure [1]. The proton SSF g_1^p has been measured in [2]; the first data on the neutron SSF were recently obtained by the E142 and SMC collaborations [3, 4]; future experiments [5] will improve the knowledge on g_1^p and g_1^n and will provide first measurements of g_2^p and g_2^n. The neutron SSF are obtained from the spin asymmetry measured in Deep Inelastic Scattering (DIS) of longitudinally polarized electrons off polarized nuclear targets, viz. $^2\vec{\mathrm{H}}$ and $^3\vec{\mathrm{He}}$. As is well known, the use of $^3\vec{\mathrm{He}}$ targets, which will be considered in this talk, is motivated by the observation that, in the simplest picture of ^{3}He (all nucleons in S wave), protons have opposite spins, so

[1] Invited talk at VI Workshop on "Perspectives in Nuclear Physics at Intermediate Energies", ICTP, Trieste, May 3-7, 1993 (World Scientific, Singapore).

that their contribution to the asymmetry largely cancels out. However, such a cancellation does not occur if other components of the three body wave function are considered; moreover, the fact that electrons scatter off nucleons having a certain momentum and energy distribution may, in principle, limit the possibility to obtain information on nucleon SSF from scattering experiments on nuclear targets. In Ref. [6], the question has been quantitatively discussed as to whether and to what extent the extraction of g_1^n from the asymmetry of the process $^3\vec{\mathrm{He}}(\vec{e},e')\mathrm{X}$ could be hindered by nuclear effects arising from small wave function components of ^{3}He, as well as from Fermi motion and binding effects on DIS. In this talk, the approach and main conclusions of Ref. [6] will be discussed, and the first theoretical predictions on the SSF g_2^3 and g_2^N will be presented. The basic nuclear ingredient used in our calculations is the spin dependent spectral function of ^{3}He [7], which allows one to take into account at the same time Fermi motion and binding corrections (unlike Ref. [8] where the $^3\vec{\mathrm{He}}$ asymmetry has been calculated taking into account S' and D waves but considering only Fermi motion and omitting Q^2 dependent terms); moreover, our work is based on a recent, improved theoretical description of inclusive scattering of polarized electrons by polarized nuclei [9].

For inclusive scattering of longitudinally polarized electrons off a polarized $J = \frac{1}{2}$ target with atomic weight A, the longitudinal asymmetry reads as follows:

$$A_{||} = \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}} = 2x[1 + R(x,Q^2)]\frac{g_1^A(x,Q^2) - \frac{Q^2}{\nu(\epsilon_1+\epsilon_2\cos\theta)} g_2^A(x,Q^2)}{F_2^A(x,Q^2)} \equiv A_{\vec{A}} \tag{1}$$

where $\sigma_{\uparrow\uparrow(\uparrow\downarrow)}$ is the differential cross section corresponding to the target spin parallel (antiparallel) to the electron spin; $x = Q^2/2M\nu$ is the Bjorken variable; g_1^A and g_2^A are the nuclear SSF; F_2^A is the spin–independent structure function of the target A; $R(x,Q^2) = \sigma_L(x,Q^2)/\sigma_T(x,Q^2)$; ϵ_1 and ϵ_2 are the energies of the incoming and outgoing electrons.

In order to extend to polarized DIS the usual convolution approach adopted to treat the unpolarized DIS [11], let us first consider the general case of inclusive scattering by spin $\frac{1}{2}$ targets in impulse approximation. For the nuclear spin structure functions g_1^A and g_2^A one gets

$$\begin{aligned} g_1^A(x,Q^2) \;=\; & \sum_N \int dE\, d\mathbf{p}\, dz \Bigg\{ \frac{1}{z} g_1^N\left(\frac{x}{z},Q^2\right) \Bigg[P_{||}^N(\mathbf{p},E) \\ & - \frac{|\mathbf{p}|}{M}\left(\frac{\nu}{|\mathbf{q}|} - \frac{|\mathbf{p}|\cos\alpha}{E_p+M}\right)\mathcal{P}^N(\mathbf{p},E)\Bigg] - \frac{Q^2}{|\mathbf{q}|^2}\frac{1}{M}\mathcal{L}^N \Bigg\} \\ & \delta\left(z - \frac{p\cdot q}{M\nu}\right) \end{aligned} \tag{2}$$

and

$$g_2^A(x,Q^2) \;=\; \sum_N \int dE\, d\mathbf{p}\, dz \frac{1}{M}\Bigg\{ \Bigg[\frac{1}{z} g_1^N\left(\frac{x}{z},Q^2\right) \frac{\nu}{|\mathbf{q}|}|\mathbf{p}|\mathcal{P}^N(\mathbf{p},E)$$

$$+\frac{1}{z^2}g_2^N\left(\frac{x}{z},Q^2\right)\left(E_pP_{||}^N(\mathbf{p},E)-\frac{|\mathbf{p}|^2\cos\alpha}{M+E_P}\mathcal{P}^N(\mathbf{p},E)\right)\Bigg]$$
$$-\frac{\nu^2}{|\mathbf{q}|^2}\mathcal{L}^N\Bigg\}\delta\left(z-\frac{p\cdot q}{M\nu}\right) \tag{3}$$

with

$$\mathcal{L}^N=\left[\frac{1}{z}g_1^N\left(\frac{x}{z},Q^2\right)\mathcal{H}_1^N+\frac{|\mathbf{q}|}{\nu}\frac{1}{z^2}g_2^N\left(\frac{x}{z},Q^2\right)\mathcal{H}_2^N\right] \tag{4}$$

$$\mathcal{H}_1^N=\left(\frac{1}{2}\right)\frac{3\cos^2\alpha-1}{\cos\alpha}\left[\frac{|\mathbf{p}|^2}{(E_p+M)}\mathcal{P}^N(\mathbf{p},E)+M\frac{P_\perp^N(\mathbf{p},E)}{\sin\alpha}\right] \tag{5}$$

$$\begin{aligned}\mathcal{H}_2^N &= |\mathbf{p}|\left[\mathcal{P}^N(\mathbf{p},E)-\frac{P_\perp^N(\mathbf{p},E)}{\sin\alpha}\right]\\ &-\frac{\nu}{2|\mathbf{q}|}\frac{3\cos^2\alpha-1}{\cos\alpha}\left[\frac{|\mathbf{p}|^2}{(E_p+M)}\mathcal{P}^N(\mathbf{p},E)-E_p\frac{P_\perp^N(\mathbf{p},E)}{\sin\alpha}\right] \quad .\end{aligned} \tag{6}$$

In the above equations the sums extend over all nucleons N; $p\equiv(p^0,\mathbf{p})$ is the four–momentum of the bound nucleon, with $p^0=M_A-[(E-M+M_A)^2+|\mathbf{p}|^2]^{\frac{1}{2}}$; E is the nucleon removal energy; $E_p=[M^2+|\mathbf{p}|^2]^{\frac{1}{2}}$; $\cos\alpha=\mathbf{p}\cdot\mathbf{q}/|\mathbf{p}||\mathbf{q}|$. The quantities $P_{||}^N(\mathbf{p},E)$, $P_\perp^N(\mathbf{p},E)$, $\mathcal{P}^N(\mathbf{p},E)$ are defined as follows [7]:

$$\begin{aligned}P_{||}^N(\mathbf{p},E) &= P^N_{\frac{1}{2}\frac{1}{2}M}(\mathbf{p},E)-P^N_{-\frac{1}{2}-\frac{1}{2}M}(\mathbf{p},E), &(7)\\ P_\perp^N(\mathbf{p},E) &= 2P^N_{\frac{1}{2}-\frac{1}{2}M}(\mathbf{p},E)e^{i\phi}, &(8)\\ \mathcal{P}^N(\mathbf{p},E) &= \sin\alpha P_\perp^N(\mathbf{p},E)+\cos\alpha P_{||}^N(\mathbf{p},E), &(9)\end{aligned}$$

where ϕ is the polar angle, and

$$\begin{aligned}P^N_{\sigma\sigma'M}(\mathbf{p},E) &= \sum_f[\langle\psi^f_{A-1};N,\mathbf{p},\sigma'|\psi_{J,M}\rangle]^*[\langle\psi^f_{A-1};N,\mathbf{p},\sigma|\psi_{J,M}\rangle]\\ &\quad \delta(E-E^f_{A-1}+E_A) \quad .\end{aligned} \tag{10}$$

is the spin dependent spectral function. Of particular relevance are the "up" and "down" spectral functions $P^N_{\frac{1}{2}\frac{1}{2}\frac{1}{2}}$ and $P^N_{-\frac{1}{2}-\frac{1}{2}\frac{1}{2}}$, respectively, for they determine the nucleon polarizations, viz. the quantities

$$\begin{aligned}P_N^{(+)} &= \int P^N_{\frac{1}{2}\frac{1}{2}\frac{1}{2}}(\mathbf{p},E)d\mathbf{p}dE &(11)\\ P_N^{(-)} &= \int P^N_{-\frac{1}{2}-\frac{1}{2}\frac{1}{2}}(\mathbf{p},E)d\mathbf{p}dE \quad . &(12)\end{aligned}$$

representing the probability to have a proton (neutron) with spin parallel (+) or antiparallel (–) to ^{3}He spin. Using in Eqs. (2) e (3) the proper nucleon SSF $g^N_{1(2)}$, the nuclear SSF $g^A_{1(2)}$ can be evaluated in the quasielastic, inelastic and DIS regions.

In the Bjorken limit ($\nu/|\mathbf{q}| \to 1$, $Q^2/|\mathbf{q}|^2 \to 0$), the nuclear asymmetry reduces to the following expression

$$A_{||} = 2x \frac{g_1^A(x)}{F_2^A(x)} \tag{13}$$

where

$$g_1^A(x) = \sum_N \int_x^A dz \frac{1}{z} g_1^N\left(\frac{x}{z}\right) G_1^N(z) \quad , \tag{14}$$

with the spin dependent light cone momentum distribution $G_1^N(z)$ given by

$$G_1^N(z) = \int dE \int d\mathbf{p} \Big\{ P_{||}^N(\mathbf{p}, E) - \left[1 - \frac{p_{||}}{E_p + M}\right] \frac{|\mathbf{p}|}{M} \mathcal{P}^N(\mathbf{p}, E) \Big\} \delta\left(z - \frac{p^+}{M}\right) \quad , \tag{15}$$

$p^+ = p^0 - p_{||}$ being the nucleon light cone momentum. It should be pointed out that if the term proportional to $\mathcal{P}^N$ in Eq. (15) is disregarded, $G_1^N(z)$ reduces to the difference between the "up", $f_N^{(+)}$, and "down", $f_N^{(-)}$, light–cone momentum distributions, viz. $G_1^N(z) = f_N^{(+)}(z) - f_N^{(-)}(z)$, where

$$f_N^{(+)}(z) \;=\; \int dE \int d\mathbf{p}\, P^N_{\frac{1}{2}\frac{1}{2}\frac{1}{2}}(\mathbf{p}, E)\, \delta\left(z - \frac{p^+}{M}\right) \tag{16}$$

$$f_N^{(-)}(z) \;=\; \int dE \int d\mathbf{p}\, P^N_{-\frac{1}{2}-\frac{1}{2}\frac{1}{2}}(\mathbf{p}, E)\, \delta\left(z - \frac{p^+}{M}\right) \quad . \tag{17}$$

Three models for ^{3}He asymmetry, in order of increasing complexity, have been considered, viz.:

1) *No nuclear effects.* One assumes that

$$g_1^3(x, Q^2) \;=\; g_1^n(x, Q^2) \tag{18}$$

$$A_{^3\vec{\mathrm{He}}} \;=\; f_n A_{\vec{n}} \tag{19}$$

where $A_{\vec{n}}(x, Q^2)$ is the neutron asymmetry and $f_n = F_2^n(x, Q^2)/[2F_2^p(x, Q^2) + F_2^n(x, Q^2)]$ the neutron dilution factor. Such a picture is equivalent to considering polarized electron scattering off $^3\vec{\mathrm{He}}$ described as a pure symmetric S wave disregarding, moreover, Fermi motion and binding effects.

2) *Proton contribution within realistic wave function of ^{3}He.* Besides the S wave, the three body wave function contains a percentage of S' and D waves, $P_{S'}$ and P_D, which are responsible for a proton contribution to the polarization of $^3\vec{\mathrm{He}}$. The amount of such a contribution can be determined by calculating the nucleon polarizations $P_N^{(\pm)}$ using the ground state wave function. In a pure S wave state $P_n^{(+)} = 1$, $P_n^{(-)} = 0$ and $P_p^{(+)} = P_p^{(-)} = \frac{1}{2}$, whereas for a three–body wave function containing S, S' and D waves, one has [12, 13]

$$P_n^{(\pm)} = \frac{1}{2} \pm \frac{1}{2} \mp \Delta, \tag{20}$$

$$P_p^{(\pm)} = \frac{1}{2} \mp \Delta' \quad , \tag{21}$$

where $\Delta = \frac{1}{3}[P_{S'} + 2P_D]$ and $\Delta' = \frac{1}{6}[P_D - P_{S'}]$. From world calculations on the three body system one obtains, in correspondence of the experimental value of the binding energy of ^{3}He, $\Delta = 0.07 \pm 0.01$ and $\Delta' = 0.014 \pm 0.002$ [12]. It is also useful to express $P_N^{(\pm)}$ in terms of the spin dependent light–cone momentum distributions [Eqs. (16) and (17)], namely

$$P_N^{(\pm)} = \int f_N^{(\pm)}(z) dz \quad . \tag{22}$$

Thus if the S' and D waves are considered and Fermi motion and binding effects are disregarded, one can write

$$g_1^3(x, Q^2) = 2p_p g_1^p(x, Q^2) + p_n g_1^n(x, Q^2) \tag{23}$$

$$A_{^3\vec{\mathrm{He}}} = 2 f_p p_p A_{\vec{p}} + f_n p_n A_{\vec{n}} \tag{24}$$

where $f_{p(n)}(x, Q^2) = F_2^{p(n)}(x, Q^2)/[2F_2^p(x, Q^2) + F_2^n(x, Q^2)]$ is the proton (neutron) dilution factor, $A_{\vec{p}(\vec{n})}(x, Q^2) = 2x g_1^{p(n)}(x, Q^2)/F_2^{p(n)}(x, Q^2)$ is the proton (neutron) asymmetry and $p_{p(n)}$ are the effective nucleon polarizations

$$p_p = P_p^{(+)} - P_p^{(-)} = -0.028 \pm 0.004 \tag{25}$$

$$p_n = P_n^{(+)} - P_n^{(-)} = 0.86 \pm 0.02 \tag{26}$$

The above values correspond to Eqs. (20) and (21), while Eq. (22) with our spin dependent spectral function yields $p_p = -0.030$ and $p_n = 0.88$.

3) *Proton contribution within the convolution approach.* The asymmetry is given by Eqs. (13), (14) and (15) in the Bjorken limit and by Eqs. (1)–(6) at finite values of Q^2.

Calculations have been performed using for the nucleon SSF g_1^N the one proposed in Ref. [14] and for the effective nucleon polarization $p_{p(n)}$, the values given by Eqs. (25) and (26).

In Fig. 1 the neutron and proton spin–dependent light cone momentum distributions in ^{3}He, [Eq. (15)] are presented. Because of the small contribution from $\mathcal{P}^N$ they are almost enterily given by the difference between the "up" and "down" light cone momentum distributions $f_N^{(\pm)}$ [Eqs. (16) and (17)], which are shown in Fig. 2. The Bjorken limit of the $^3\vec{\mathrm{He}}$ asymmetry [Eq. (13)] is presented in Fig. 3(a). It can be seen that the proton contribution is very important at $x \geq 0.3$, and therefore it hinders in principle the extraction of the neutron structure function. In order to understand how much of the proton contribution to the asymmetry is given by S' and D waves, by Fermi motion and by binding effects, the asymmetry predicted by the convolution approach is compared in Fig. 3(b) with the predictions of the models 1 and 2 [Eqs. (18)–(24)]. It can be seen that the asymmetry which totally lacks of nuclear effects (binding and Fermi motion as well as S' and D waves), strongly differs from the ones which include these

effects; however it can also be seen that at least for $x \leq 0.9$ nuclear effects can reliably be taken care of by Eq. (24), i.e. by considering only the effective nucleon polarization induced by S' and D waves. If so, the neutron asymmetry $\tilde{A}_{\vec{n}}$ can be obtained from the ^{3}He asymmetry using Eq. (24), viz.

$$\tilde{A}_{\vec{n}}(x) = \frac{1}{f_n p_n} \left[A_{^3\vec{He}}(x) - 2 p_p f_p A_{\vec{p}}(x) \right] \quad . \tag{27}$$

In Fig. 4(a), $\tilde{A}_{\vec{n}}$ calculated from Eq. (27) using the convolution formula for $A_{^3\vec{He}}(x)$ is compared with the free neutron asymmetry; it can be seen that the two quantities are very close to each other, differing, because of binding and Fermi motion effects, by at most 4% for $x \leq 0.9$. The same comparison for the SSF is presented in Fig. 4(b), where the free neutron SSF is compared with the quantity:

$$\tilde{g}_1^n(x) = \frac{F_2^n(x) \tilde{A}_{\vec{n}}(x)}{2x} \quad . \tag{28}$$

It has been shown in Ref. [6] that the small effects from binding and Fermi motion can be understood by expanding $\frac{1}{z} g_1^N \left(\frac{x}{z} \right)$ in Eq. (14) around $z = 1$ and by disregarding the term proportional to $\mathcal{P}^N$ in Eq. (15); by this way the correctness of Eq. (23) can be easily justified.

It appears therefore that the only relevant nuclear effects in inclusive DIS of polarized electrons off polarized ^{3}He in the Bjorken limit are those related to the proton and neutron effective polarizations arising from S' and D waves; such a result, moreover, does not seem to crucially depend upon the form of g_1^N.

In order to perform a significant comparison between theoretical predictions and experimental data, one has to investigate the Q^2 dependence of $g_1^3(x, Q^2)$ by evaluating Eq. (2). Taking [15]

$$g_2^N(x, Q^2) = -g_1^N(x, Q^2) + \int_x^1 dy \frac{1}{y} g_1^N(y, Q^2) \tag{29}$$

and assuming the kinematics of Ref. [4], we found that $g_1^3(x, Q^2)$ and $g_1^3(x)$ differ by at most 15%. These are preliminary results and it is not the aim of this talk to discuss their possible implications on the Bjorken sum rule.

Future experiments are planning the direct measurement of g_2^n and g_2^3; the first quantity [15] is shown in Fig. 5(a) whereas g_2^3 calculated taking into account the proton and neutron effective polarizations, i.e. using the expression resulting from the series expansion of $\frac{1}{z^2} g_2^N \left(\frac{x}{z} \right)$ around $z = 1$ in Eq. (3), is presented in Fig. 5(b).

A detailed analysis of the transverse asymmetry, as well as of the Q^2 dependent effects, is in progress and will be reported elsewhere.

Captions

FIG. 1. (a) The nucleon spin–dependent light–cone momentum distributions $G_1^N(z)$ in ^{3}He [Eq. (15)] for the neutron (a) and the proton (b), respectively.

FIG. 2. (a) The "up" and "down" light–cone momentum distributions $f_N^{(+)}(z)$ [Eq. (16)] (full) and $f_N^{(-)}(z)$ [Eq. (17)] (dashed) for the neutron (a) and the proton (b) in ^{3}He.

FIG. 3. (a) The ^{3}He asymmetry [Eq. (1)] calculated within the convolution approach [Eq. (14)](full). Also shown are the neutron (short–dashed) and proton (long–dashed) contributions. (b) The ^{3}He asymmetry calculated within different nuclear models. Dotted line: no nuclear effects [Eq. (19)]; short–dashed line: S' and D waves of ^{3}He taken into account [Eq. (24)]; long–dashed line: S' and D waves of ^{3}He taken into account plus Fermi motion effects; full line: S' and D waves of ^{3}He taken into account plus Fermi motion and binding effects [after [6]].

FIG. 4. (a) The free neutron asymmetry (dots) compared with the neutron asymmetry given by Eq. (27)(full). (b) The same as in (a) but for the SSF g_1^n. The dotted line represents the free neutron structure function g_1^n, whereas the full line represents the neutron structure function given by Eq. (28). The difference between the two curves is due to Fermi motion and binding effects [after [6]].

FIG. 5. (a) The free neutron structure function g_2^n [15];(b) the ^{3}He structure function g_2^3 calculated taking into account the effective proton and neutron polarizations (full); the neutron and proton contribution are represented by the short–dashed and long–dashed curves, respectively.

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